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Coupled Langmuir and nonlinear ion-acoustic waves in collisional plasmas

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It is shown that the interaction of Langmuir waves and finite-amplitude collision-dominated ion-acoustic waves in a collisonal plasma is governed by the coupling of a damped nonlinear Schrödinger equation and a driven Korteweg–de Vries–Burger equation. The system differs considerably from the Zakharov and related systems for a collisionless plasma.

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I. INTRODUCTION

Processes in low-temperature collisional plasmas have been of much recent interest because of their relevance in many laboratory plasmas as well as many modern technological applications of plasmas. Collisional plasmas have the property that dissipative effects and the corresponding transport are important, and the charged particles have comparable temperatures. An important problem in collisional plasmas is that of wave propagation, as the stability and the subsequent nonlinear behavior of the waves can crucially affect the property of the plasma [2–6].

Two commonly occurring modes in collisional plasmas are the Langmuir and the ion-acoustic waves. The high-frequency ($\omega \approx \omega_{pe} \gg \nu_e$, where ω_{pe} is the electron plasma frequency and ν_e is an effective electron collision frequency) Langmuir waves are driven by the ambipolar electrostatic force and electron inertia, and the low-frequency ($\omega \approx kc_s \ll \nu_i$, where k is the wave vector, c_s is the ion sound speed, and ν_i is an effective ion collision frequency) collisional ion-acoustic waves are driven by the electron pressure and ion inertia, the coupling between the species being maintained by electrostatic forces and electron-ion collisions. That is, the electron dynamics in the Langmuir waves can be described by the usual warm-fluid theory modified by weak frictional dissipation, while both the electron and ion dynamics in the ion-acoustic waves can be described by the Braginskii [1] transport equations. Thus, although aside from the damping terms, the linear dispersion relations for both waves remain similar to that of the collisionless hot-plasma case, the physics of the ion-acoustic waves in a highly collisional plasma is more complicated, since both electrostatic and collisional effects enter into play. In particular, collisons between the unlike particles can also couple the dynamics of the ions and the electrons. In other words, the waves involve both plasma and neutral-fluid properties.

The nonlinear interaction of Langmuir and ion-acoustic modes has been of great interest in plasma physics for several decades [7–13]. The problem is important not only because of the frequent occurrence of the two modes, but also because the interaction is representative of many similar wave interactions in which finite amplitude high-frequency waves can modulate the background plasma parameters such as the density, magnetic field, etc. through low-frequency waves or quasimodes. Such interactions have been thought to be the cause of turbulence in many plasmas. Although there exists a large number of studies on the Langmuir-wave–ion-acoustic wave interaction in collisionless or weakly dissipative plasmas, there seems to be no comprehensive investigation of such interactions in a collision-dominated plasma. In this paper, we derive the equations governing the coupling of Langmuir and ion-acoustic waves in a collisional plasma, using the complete set of the Braginskii equations [1] for the low-frequency motion. Ion nonlinearity is taken into account since it is well known that the ponderomotive force of the high-frequency waves can easily cause finite-amplitude density modifications [7,14,15]. We found that the interaction is described by a coupled set of evolution equations consisting of a nonlinear Schrödinger equation (NSE) modified by weak dissipation, and a high-frequency field-driven Korteweg–de Vries-Burger (KdVB) equation. These equations are the counterpart of the Zakharov [8,9] or more precisely, the Nishikawa *et al.* [7], equations for a collisionless plasma. Here, besides the appearance of dissipative terms, the scaling and therefore the physics in these equations are different from those of the collisionless case. In particular, in contrast to the latter, where the nonlinearity originates from ion convection and electron pressure, here it is dominated by the thermal forces and interspecies heat transfer. Furthermore, the driven KdVB equation is not reducible to a KdV equation because of the collisional scaling. Quasistationary envelope shocklike solutions are obtained numerically.

II. BASIC EQUATIONS

For the Langmuir waves, we use the warm-fluid equations for the electrons. Since the Langmuir frequency is much larger than any of the electron collision frequen-

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cies, collisional damping can be taken into account by including a simple frictional damping term $-\nu_e \mathbf{v}_e$ (where ν_e and $\mathbf{v}_e$ are the effective electron collision frequeny and the electron fluid velocity, respectively) on the right hand side of the electron momentum equation.

The frequency of the collision-dominated ion-acoustic waves is much less than the electron and ion collision frequencies. Thus for both species one can use the Braginskii transport equations [1], which were obtained from the kinetic equation with the Landau collision integral. We are interested in the case of strong Langmuir fields, namely $|E^L|^2/n_0T \gg m_e/m_i$ and $|E^L|^2/n_0T \geq (1 - V^2/v_s^2)^2 \gg m_e/m_i$, where E^L is the Langmuir wave electric field, n_0 and T are the unperturbed density and temperature, m_e/m_i is the electron to ion mass ratio, V is the wave speed, and v_s is the sound speed. Thus we have to take into account the nonlinear as well as the dispersive and dissipative terms in the derivation of the low-frequency potential. Furthermore, unlike the case for hot collisionless plasmas, where for the ion waves the electrons are in thermal equilibrium and governed by the Boltzmann distribution, here the full dynamics of both the ions and electrons must be taken into account.

Accordingly, for the ion waves we start with the complete set of the Braginskii's equations for both the electrons and ions. For the fluid velocities $\mathbf{v}_e$ and $\mathbf{v}_i$, of the electrons and ions, we have

$$m_e n_e(\partial_t + \mathbf{v}_e \cdot \nabla)v_{e;j} = -\nabla_j n_e T_e - \nabla_l \pi_{lj}^{(e)} - en_eE_j + R_j \,, \quad (1)$$

and

$$m_i n_i(\partial_t + \mathbf{v}_i \cdot i\nabla)v_{i;j} = -\nabla_j n_i T_i - \nabla_l \pi_{lj}^{(i)} + en_iE_j - R_j \,, \quad (2)$$

where the sub- or superscripts e and i denote electron and ion quantities, respectively, and l and j are dummy spatial-direction indices. Furthermore, $\pm e$, $m_{e,i}$, $n_{e,i}$, and $T_{e,i}$ are the charge, mass, density, and temperature of the electrons and ions, and $\mathbf{E}$ is the total electric field.

Equations (1) and (2) are closed by the continuity equations

$$\partial_t n_{e,i} + \nabla \cdot (n_{e,i}\mathbf{v}_{e,i}) = 0 \,, \quad (3)$$

and the energy balance equations

$$\tfrac{3}{2}n_{e,i}(\partial_t + \mathbf{v}_{e,i} \cdot \nabla)T_{e,i} + n_{e,i}T_{e,i}\nabla \cdot \mathbf{v}_{e,i} = -\nabla \cdot \mathbf{q}^{(e,i)} - \pi_{lj}^{(e,i)}\nabla v_{e,i;l} + Q_{e,i} \,. \quad (4)$$

In the above equations, the terms $-\nabla nT_{e,i}$ are the pressure forces of the electron and ion fluids. The stress tensors $\pi_{lj}^{(e,i)}$ are given by

$$\pi_{lj}^{(e)} = -0.73\frac{n_eT_e}{\nu_e}w_{lj}^{(e)} \,, \; \pi_{lj}^{(i)} = -0.96\frac{n_iT_i}{\nu_i}w_{lj}^{(i)} \,, \quad (5)$$

where ν_j is the collision frequency of species j, and the rate of strain tensors $w_{lj}^{(e,i)}$ is defined by

$$w_{lj}^{(e,i)} = bf\nabla_j v_{e,i;l} + \nabla_l v_{e,i;j} - \tfrac{2}{3}\delta_{lj}\nabla \cdot \mathbf{v}_{e,i} \,. \quad (6)$$

The frictional force $\mathbf{R}$ between the electrons and ions is

$$\mathbf{R} = \mathbf{R_u} + \mathbf{R}_T \,, \quad (7)$$

where $\mathbf{R_u}$ is associated with the force of relative friction (for $\omega \ll \nu_e$)

$$\mathbf{R_u} = -0.51 n_e m_e \nu_e \mathbf{u} \,, \quad (8)$$

which depends only on the relative velocity $\mathbf{u} = \mathbf{v}_e - \mathbf{v}_e$ between the electrons and ions. In the opposite limit, namely $\omega \gg \nu_e$, we use the *mathematically* similar relation $\mathbf{R_u} \simeq -n_e m_e \nu_e \mathbf{u}$.

The thermal-gradient frictional force $\mathbf{R}_T$ apppearing in (7) is given by

$$\mathbf{R}_T = -0.71 n_e \nabla T_e \,. \quad (9)$$

Furthermore, the heat fluxes $\mathbf{q}^{(e,i)}$ are

$$\mathbf{q}^{(e)} = \mathbf{q}_{\mathbf{u}}^{(e)} + \mathbf{q}_T^{(e)} = 0.71 n_e T_e \mathbf{u} - 3.16\frac{n_eT_e}{m_e\nu_e}\nabla T_e$$

$$\mathbf{q}^{(i)} = -3.9\frac{n_iT_i}{m_i\nu_i}\nabla T_i \,, \quad (10)$$

and the heating powers $Q_{e,i}$ are

$$Q_e = -\mathbf{R} \cdot \mathbf{u} - Q_i \,, \quad Q_i = 3\frac{m_e}{m_i}n_e\nu_e(T_e - T_i) \,. \quad (11)$$

In the following, we shall solve the above equations by expanding in powers of the electric field $\mathbf{E}$.

III. EQUATION FOR HIGH-FREQUENCY WAVES

To obtain the evolution equation for the Langmuir wave envelope, we must take into consideration electric fields of the low-frequency sound waves (which modulate the density) as well as the high-frequency Langmuir waves. The derivation follows standard methods [2,3,5–13], the difference being the inclusion of the collision terms. Thus, for the Fourier components

$$\mathbf{E}_{\mathbf{k},\omega} = \frac{1}{(2\pi)^4}\int \mathbf{E}(\mathbf{r},t)\exp(i\omega t - i\mathbf{k}\cdot\mathbf{r})d\mathbf{r}\,dt \quad (12)$$

of the longitudinal fields

$$\mathbf{E}_{\mathbf{k},\omega} = \frac{\mathbf{k}}{|\mathbf{k}|}E_{\mathbf{k},\omega}, \quad (13)$$

we obtain

$$\varepsilon_{\mathbf{k},\omega}^L E_{\mathbf{k},\omega}^L = -\frac{4\pi i}{|\mathbf{k}|}\rho_{\mathbf{k},\omega}^{LS} \,, \quad (14)$$

where $\varepsilon^L_{\mathbf{k},\omega}$ is the usual linear (high-frequency) dielectric permittivity

$$\varepsilon^L_{\mathbf{k},\omega} = 1 - \frac{\omega_{pe}^2}{(\omega + i\nu_e)\omega} - \frac{3\mathbf{k}^2 v_{Te}^2 \omega_{pe}^2}{(\omega + i\nu_e)^3\omega} , \tag{15}$$

$\omega_{pe} = (4\pi n_e e^2/m_e)^{1/2}$ is the electron plasma frequency and $v_{Te} = (T_0/m_e)^{1/2}$ is the electron thermal velocity. Note that ν_e describes weak (since $\nu_e \ll \omega_{pe}$) collisional damping of the Langmuir waves.

The nonlinear second-order charge density proportional to the high-frequncy Langmuir and low-frequency sound fields (recall that the low-frequency quantities are governed by the Braginskii equations) is

$$\rho^{LS}_{\mathbf{k},\omega} = -\frac{ie}{4\pi m_e} \int \frac{\omega_{pe}^2 (\mathbf{k}\cdot\mathbf{k}_1)|\mathbf{k}_2|}{\omega\omega_1\omega_2\kappa\omega_e|\mathbf{k}_1|} E^L_{\mathbf{k}_1,\omega_1} E^S_{\mathbf{k}_2,\omega_2} d\mathbf{K}\, d\Omega , \tag{16}$$

where the subscripts 1 and 2 are dummy wave number indices, S denotes the sound wave field, and $d\Omega\, d\mathbf{K}$ stands for $\delta(\omega-\omega_1-\omega_2)\delta(\mathbf{k}-\mathbf{k}_1-\mathbf{k}_2)\, d\omega_1 d\omega_2 d\mathbf{k}_1 d\mathbf{k}_2$. In Eq. (16) we have defined

$$\kappa = 1 + \left(0.51\nu_e + 1.71\frac{k^2 v_{Te}^2}{\Omega_e}\frac{1.71 - 0.71\Delta_i}{\Delta}\right) \times \left(\frac{1}{\omega_e} + \frac{m_e}{m_i}\frac{1}{\omega_i}\right) \approx 1 , \tag{17}$$

$$\omega_e = -i\omega + i\frac{k^2 v_{Te}^2}{\omega}\left(1 - 1.71\frac{i\omega\Delta_e}{\Omega_e\Delta}\right) + \frac{4}{3}0.73\frac{k^2 v_{Te}^2}{\nu_e} \approx i\frac{k^2 v_{Te}^2}{\omega}\frac{5 + 2\cdot 0.71}{3} , \tag{18}$$

$$\omega_i = -i\omega + i\frac{k^2 v_{Ti}^2}{\omega}\left(1 - \frac{i\omega\Delta_e}{\Omega_e\Delta} + 0.71\frac{i\omega\Delta_e}{\Omega_e\Delta}\right) + \frac{4}{3}0.96\frac{k^2 v_{Te}^2}{\nu_e} \approx 7 - i\omega\left(1 - \frac{k^2 v_{Ti}^2}{\omega^2}\frac{5 - 2\cdot 0.71}{3}\right) , \tag{19}$$

$$\Omega_e = -\frac{3}{2}i\omega + 3.16\frac{k^2 v_{Te}^2}{\nu_e} + 3\frac{m_e}{m_i}\nu_e \approx 3\frac{m_e}{m_i}\nu_e , \tag{20}$$

$$\Omega_i = -\frac{3}{2}i\omega + 3.9\frac{k^2 v_{Ti}^2}{\nu_i} + 3\frac{m_e}{m_i}\nu_e \approx 3\frac{m_e}{m_i}\nu_e \approx \Omega_e , \tag{21}$$

$$\Delta_{e,i} = 1 + 3\frac{m_e}{m_i}\frac{\nu_e}{\Omega_{i,e}} \approx 2 , \tag{22}$$

and

$$\Delta = 1 - \left(3\frac{m_e}{m_i}\nu_e\right)^2 \frac{1}{\Omega_e\Omega_i} \approx -\frac{m_i}{m_e}\frac{i\omega}{\nu_e} . \tag{23}$$

Note that $T_0 = T_e \approx T_i$ is the equilibrium plasma temperature. Here and in the following, we retain the numerical coefficients in their original forms, so that the origins of the various, in particular the coupling, terms can be identified easily.

Thus, for the slowly varying amplitude $E_L = E_L(x,t)$ of the one-dimensional Langmuir waves propagating in the x direction

$$E^L = E_L \exp(-i\omega_{pe}t) + \text{c.c.} , \tag{24}$$

we obtain the damped NSE for the Langmuir waves

$$\left(i\partial_t + \frac{i\nu_e}{2} + \frac{3v_{Te}^2}{2\omega_{pe}}\partial_{xx}\right) E_L = \frac{3\beta}{20}\frac{e\omega_{pe}}{T_0} E_L \varphi_S , \tag{25}$$

where φ_S is the low-frequency acoustic wave potential ($E_S = -\partial_x\varphi_S$), and $\beta = 10/(5 + 2\cdot 0.71)$. Since for the case considered we have $|\partial_t E_L| \ll |\nu_e E_L|$, the term $\partial_t E_L$ is an order smaller than $\nu_e E_L$.

Note that the coupling coefficient on the right hand side of (25) originates from the pressure and thermal forces, as well as the heating power, and in spite of the mathematical similarity, differs considerably from that for collisionless (or weakly collisional) plasmas (compare with Refs. [7,10,11]). We also recall that, similar to the standard NSE for collisionless Langmuir waves, although the first term in (25) is small, it must be retained in order to allow for a small phase shift of the nonlinear waves, needed for balancing the complex part of the equation.

IV. EQUATION FOR LOW-FREQUENCY WAVES

For the Fourier components of the low-frequency electric fields, we have an equation similar to (14), namely

$$\varepsilon^S_{\mathbf{k},\omega} E^S_{\mathbf{k},\omega} = -\frac{4\pi i}{|\mathbf{k}|}\left(\rho^{SS}_{\mathbf{k},\omega} + \rho^{LL}_{\mathbf{k},\omega}\right) , \tag{26}$$

where the linear dielectric permittivity $\varepsilon^S_{\mathbf{k},\omega}$ for the collisional acoustic waves is given by (see [5,16])

$$\varepsilon^S_{\mathbf{k},\omega} = 1 + \frac{i\omega_{pe}^2}{\kappa\omega\omega_e} + \frac{i\omega_{pe}^2}{\kappa\omega\omega_e} \simeq \frac{i\omega_{pe}^2}{\kappa\omega_e\omega_i}\left(\omega_i + \frac{m_e}{m_i}\omega_e\right) . \tag{27}$$

The nonlinear part of (26), in contrast to (14), consists of two parts. The first, $\rho^{SS}_{\mathbf{k},\omega}$, is from the ion nonlinearity [16], and is given by

$$\rho^{SS}_{\mathbf{k},\omega} = \frac{1}{3}\frac{ie\beta^2}{4\pi m_i}\int \frac{\omega_{pi}^2\mathbf{k}^2}{\omega\omega_1\omega_2\kappa\omega_i}\left(1 + 3\frac{\mathbf{k}_1\cdot\mathbf{k}_2}{|\mathbf{k}_1||\mathbf{k}_2|}\right) \times E^S_{\mathbf{k}_1,\omega_1} E^S_{\mathbf{k}_2,\omega_2} d\mathbf{K} d\Omega . \tag{28}$$

The second nonlinear term in (26) is the generalized ponderomotive force exerted by the Langmuir waves

$$\rho^{LL}_{\mathbf{k},\omega} = -\frac{ie}{4\pi m_i}\int \frac{\omega_{pe}^2\nu_e\Delta_e\mathbf{k}^2\mathbf{k}_1\cdot\mathbf{k}_2}{\omega\omega_1\omega_2\kappa\omega_i\Omega_e\Delta|\mathbf{k}_1||\mathbf{k}_2|} \times E^L_{\mathbf{k}_1,\omega_1} E^{*,L}_{\mathbf{k}_2,\omega_2} d\mathbf{K}\, d\Omega , \tag{29}$$

where the asterisk stands for the complex conjugate. Note also that $\omega_2 \approx -\omega_{pe}$.

Finally, for the one-dimensional low-frequency field potential

$$\left[\partial_t + v_s\partial_x - \frac{3.16}{50}\frac{m_i}{m_e}\frac{v_s^2}{\nu_e}\partial_{xx} + 5\left(\frac{3.16}{50}\right)^2\left(\frac{m_i}{m_e}\right)^2\frac{v_s^3}{\nu_e^2}\partial_{xxx}\right]\varphi_S$$

$$= -\frac{4\beta}{3}\frac{e}{m_i v_s}\partial_x(\varphi_S)^2 + \frac{1}{3\beta}\frac{e\nu_e}{m_e\omega_{pe}^2}|E_L|^2 , \quad (30)$$

where $v_s = \sqrt{10T_0/3m_i}$ is the speed of the collisional ion-acoustic waves (see Refs. [5,16]). Equation (30) generalizes the standard KdVB equation for describing the nonlinear evolution of collisional ion-acoustic wave [16] in the presence of Langmuir oscillations.

Without the driving force (i.e., when $E_L = 0$), (30) has been extensively studied in the literature, and has the well known quasistationary solution depicting a shock wave with a spatially decaying and oscillating tail or front [2–4]. Unlike the KdVB equation for a weakly collisional plasma, (30) cannot be reduced to a simple KdV equation because here the scaling (i.e., the normalization parameters) is fixed, as is evident from the absence of free parameters which can be rescaled in order to neglect the dissipation term.

V. QUASISTATIONARY SOLUTION

The coupled set of nonlinear evolution equations (25) and (30) can be conveniently normalized such that in the dimensionless variables we have

$$(i\alpha\partial_t + i\gamma + \partial_{xx})\,\mathcal{E} = \mu\phi\mathcal{E} , \quad (31)$$

and

$$(\partial_t + \partial_x - \partial_{xx} + 5\partial_{xxx})\,\phi = -\partial_x(\phi)^2 + |\mathcal{E}|^2 , \quad (32)$$

where t and x have been normalized by $3.16m_i/50m_e\nu_e$ and $3.16m_i v_s/50m_e\nu_e$, respectively, and we have defined $\phi = 4\beta e\varphi/3m_i v_s^2$, and $\mathcal{E} = 2eE_L/(3m_e\omega_{pe}v_s\sqrt{50/3.16})$. We also have assumed for convenience that $\nu_e m_e \ll \nu_i m_i$ [16]. The dimensionless parameters α, γ, and μ in (31) and (32) are

$$\alpha = \frac{2}{3}\left(\frac{10}{3}\right)\frac{3.16}{50}\frac{\omega_{pe}}{\nu_e} , \quad \gamma = \frac{4}{3}\frac{\nu_e}{\omega_{pe}}\mu ,$$

$$\mu = \frac{1}{4}\left(\frac{10}{3}\right)\left(\frac{3.16}{50}\right)^2\frac{m_i}{m_e}\frac{\omega_{pe}^2}{\nu_e^2} . \quad (33)$$

We emphasize that the present scaling is completely different from that for a collisionless plasma. In particular, we note that (32) contains no free parameters.

Equations (31) and (32) describe Langmuir wave propagation or turbulence in the presence for low-frequency *finite-amplitude* ion (or background plasma) response in a highly collisional plasma. A detailed investigation of possible phenomena, especially those related to Langmuir turbulence, associated with these coupled partial differential equations is beyond the scope of this paper. Instead, we shall present a numerical solution which depicts a possible quasistationary state of the collisional Langmuir-acoustic interaction.

Assuming $\mathcal{E} = \mathcal{E}_0(\xi)\exp(-\gamma t/\alpha - i\Omega t + iKx)$, and $\varphi = \varphi(\xi)$, where $\xi = x - Vt$ with V a constant, we obtain from (31) and (32) the ordinary differential equations

$$d_{\xi\xi}\mathcal{E}_0 + \delta\mathcal{E}_0 = \mu\phi\mathcal{E}_0 , \quad (34)$$

and

$$[5d_{\xi\xi\xi} - d_{\xi\xi} + (1-V)d_\xi]\,\phi = -d_\xi(\phi)^2 + \mathcal{E}_0^2 , \quad (35)$$

where $K = (\alpha V + \gamma)/2$ and $\delta = -K^2 - \alpha\Omega$. To preserve the ordering of (31), the condition $|\gamma\mathcal{E}_0 + \alpha V d_\xi\mathcal{E}_0| \ll |\gamma\mathcal{E}_0|$ must be satisfied. From the latter we see that V should be negative and sufficiently large.

For given initial values, Eqs. (34) and (35) yields oscillating (among others) solutions for $\mathcal{E}_0$ and ϕ which increase in magnitudes, until a stage of exponential growth occurs. The rates of amplitude increase depend on the parameters δ, μ, and V. Envelope shock solutions can be constructed in a manner similar to that for the usual oscillating shock solutions of the KdVB and related equations [2,4,17]. We first note that the constants $\mathcal{E}_0 = 0$ and $\phi = \text{const}$ also satisfy the equations. Thus one needs to find solutions for $\mathcal{E}_0$ and ϕ such that at some ξ, $\mathcal{E}_0$, $d_\xi\mathcal{E}_0$, and $d_\xi\phi$ all vanish. At this point, one can match the two sets of solutions. Such a solution is given in Fig. 1, for which $\delta = 2.5$, $\mu = 0.15$, and $V = -4.5$. These parameters are chosen such that the corresponding result most clearly demonstrates the matching process and the behavior of the solutions, rather than for any specific physical application. For the latter, the amplitudes would be much smaller and the coupling parameter μ somewhat larger. At about $\xi = 17$, the matching conditions are

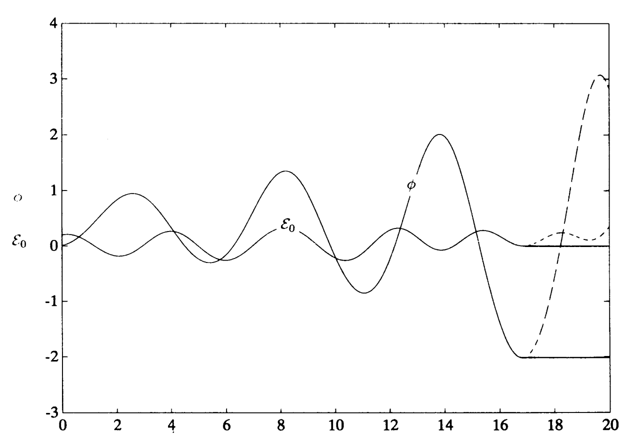

FIG. 1. A typical envelope shock wave solution obtained by matching. The dashed curves represent solution parts which are not used.

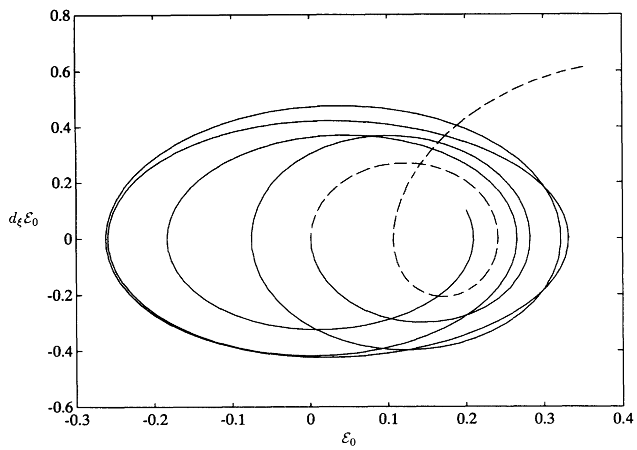

FIG. 2. The phase diagram of the Langmuir wave envelope $\mathcal{E}_0$.

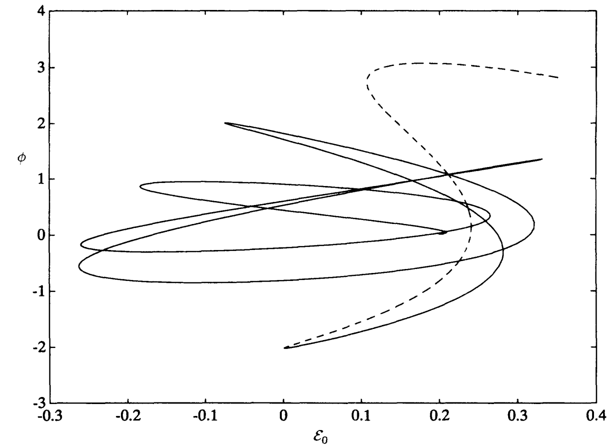

FIG. 4. The ϕ vs. $\mathcal{E}_0$ phase diagram. In the search for matchable solutions, one looks for points like $(-2, 0)$ here.

satisfied, and one can join the oscillating solutions to the solutions $\mathcal{E}_0 = 0$ and $\phi \approx -2$. Figures 2 and 3 show the phase diagrams for $\mathcal{E}_0$ and ϕ. Since the starting amplitudes are small and μ is small, one sees that the Langmuir wave envelope and the potential of the nonlinear sound waves are weakly coupled, until the nonlinear interaction becomes strong enough for the waves to affect each other. Figure 4 shows the $\mathcal{E}_0$ vs. ϕ phase diagram, which is useful in the numerical search for the existence of the suitable point at which solution matching can be made. Similar to most solutions obtained by matching [17], the higher order derivatives are not continuous, so that the solutions here are mathematically weak solutions. Oscillatory shocklike solutions which are continuous in all the derivatives may still exist. However, these are difficult to find numerically as one has to have the exactly correct set of the initial conditions. Finally, we mention that solutions in which the matching occurs at maximum values of ϕ can also be obtained.

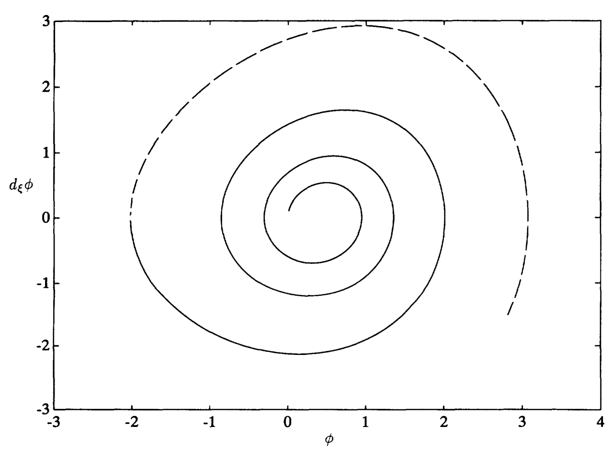

FIG. 3. The phase diagram of the acoustic-wave potential ϕ.

VI. DISCUSSION

Equations (25) and (30) describe the coupling of weakly (compared to the plasma frequency, not the rate of modulation) damped Langmuir and the low-frequency ($kc_s \ll \nu_i \ll \nu_e \ll \omega_{pe}$) nonlinear collisional ion sound waves. This coupled set of nonlinear evolution equations is the collisional counterpart of the Nishikawa *et al.* system [7] for collisionless plasmas. It governs the behavior of Langmuir turbulence in unmagnetized collisional plasmas.

Equation (32), which does not contain any dimensionless parameters, is a forced KdV equation. The corresponding quasistationary solutions, namely, the shocklike envelope structures with oscillating tails or fronts, occur in strongly collisional plasma in general. In fact, envelope solitons of the Zakharov [8,9] or Nishikawa *et al.* [7] types cannot appear in such plasmas. The fact that thermal forces and interparticle heat transfer dominate the nonlinear coupling is to be expected, since here the collisional transport is more important than that from inertia. Note also that the stationary shocklike structures found here are not true shock waves in the classical sense, because steep gradients of the physical quantities are not involved.

Finally, let us briefly compare the present study to the recent work by Goldman *et al.* [13], who considered the coupling of Langmuir and ion sound waves (or quasimodes) in a collisionless plasma. They introduced model electron moment equations which are numerically consistent with the corresponding kinetic theory results in order to obtain a more complete description of the effect of the Langmuir waves on the low-frequency dynamics. The effect shows up as an electron pressure force acting in conjunction with the usual ponderomotive force. This pressure is in turn governed by an energy equation with a linear collisionless heat flux (see also Ref. [18]), as well as a heat source from the beating of the high-frequency waves. One sees that the physics involved is in some sense similar to that of the corresponding low-frequency motion in the present paper, except that in the latter the ion nonlinearity is also included and the heat flux and heat source (which dominate the coupling) originate from actual particle-particle collisions.

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